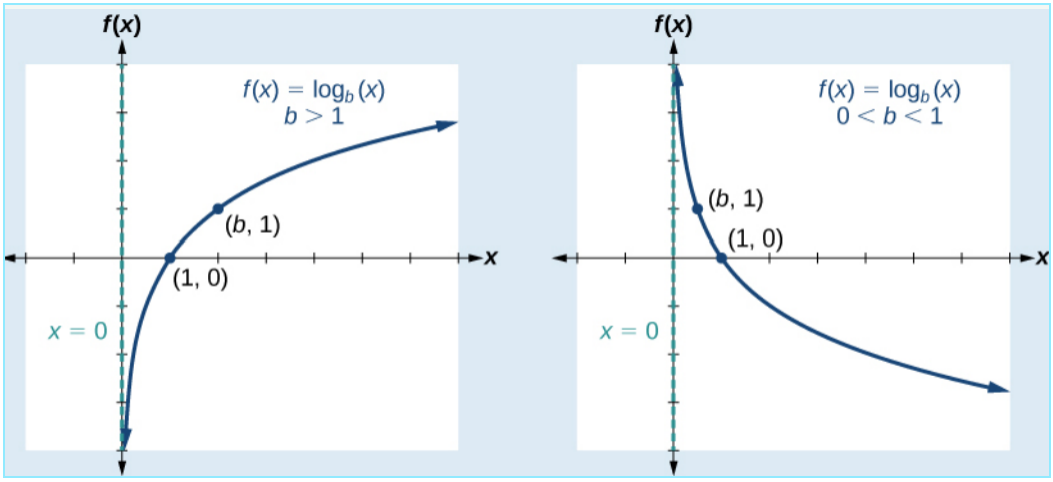
Graphs of Logarithmic Functions

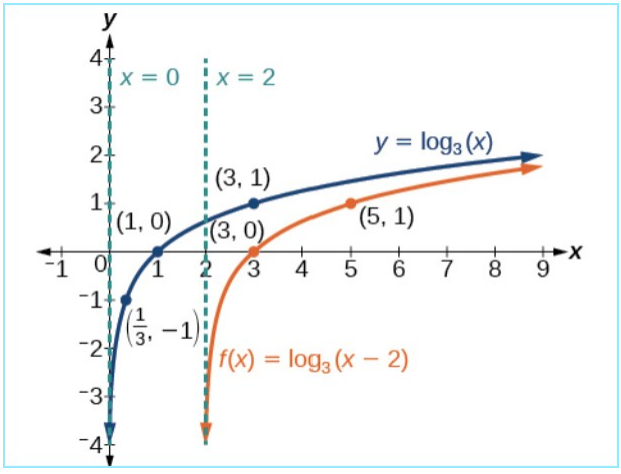
Key Points:

- The graph of the parent function $f(x) = \log_b x$ has an x −intercept at $(1,0)$, key point $(b, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$, vertical asymptote $x = 0$ and

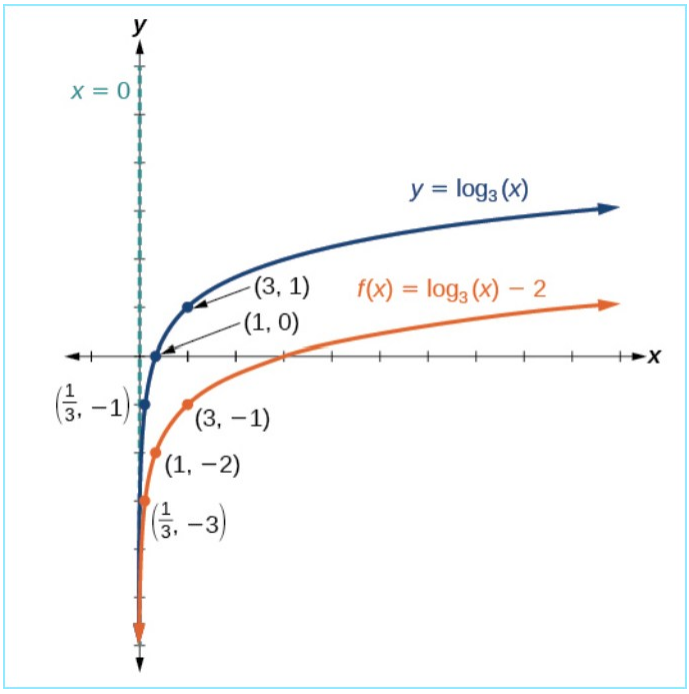
  ◦ if $b > 1$, the function is increasing.
  ◦ if $0 < b < 1$, the function is decreasing



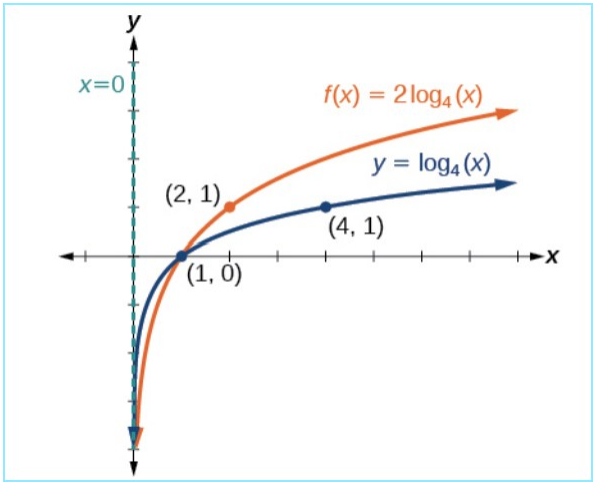
- The equation $f(x) = \log_b(x + c)$ shifts the parent function $y = \log_b(x)$ horizontally
  ◦ left c units if $c > 0$.
  ◦ right c units if $c < 0$.



- The equation $f(x) = \log_b(x) + d$ shifts the parent function $y = \log_b(x)$ vertically
  - up d units if $d > 0$.
  - down d units if $d < 0$.

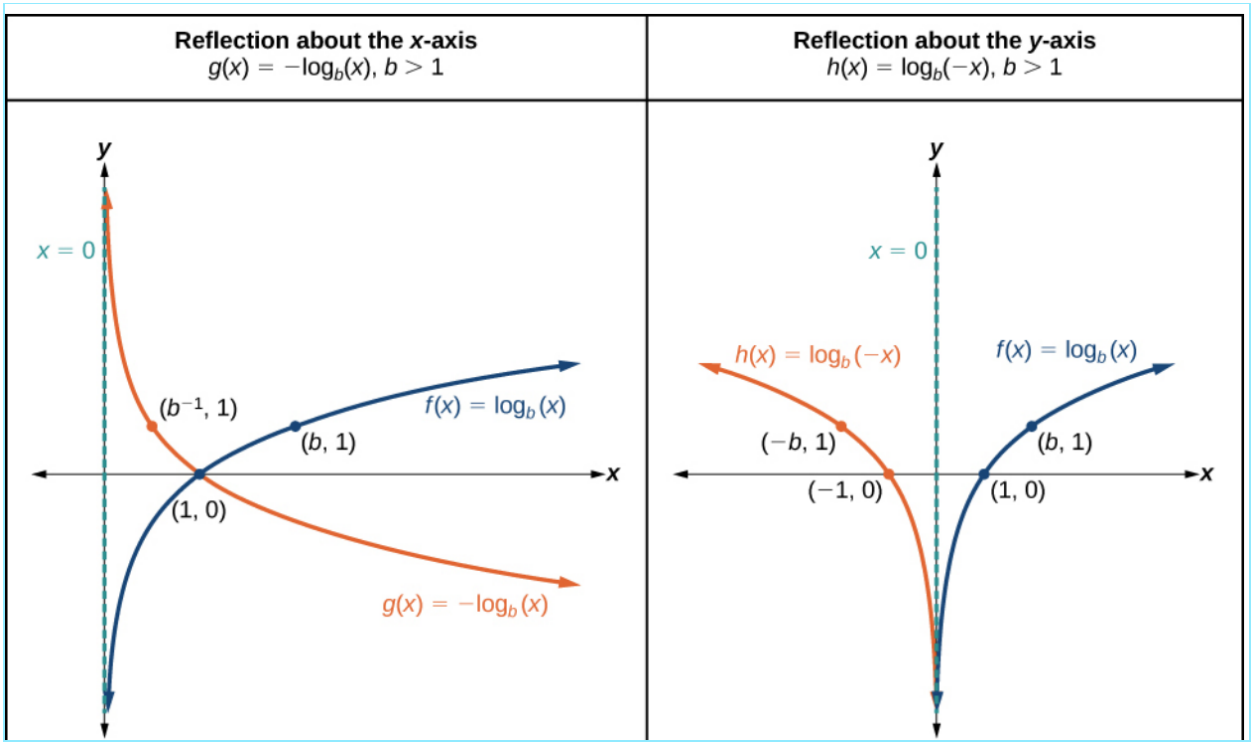


- For any constant $a > 0$, the equation $f(x) = a \log_b(x)$
  - stretches the parent function $y = \log_b(x)$ vertically by a factor of a if $|a| > 1$ meaning if $a > 1$ or $a < -1$.
  - compresses the parent function $y = \log_b(x)$ vertically by a factor of a if $|a| < 1$, meaning if $-1 < a < 1$



- The equation $g(x) = -\log_b(x)$represents a reflection of the parent function $f(x) = \log_b(x)$ about the x-axis.

- The equation $h(x) = \log_b(-x)$ represents a reflection of the parent function $f(x) = \log_b(x)$about the y-axis.



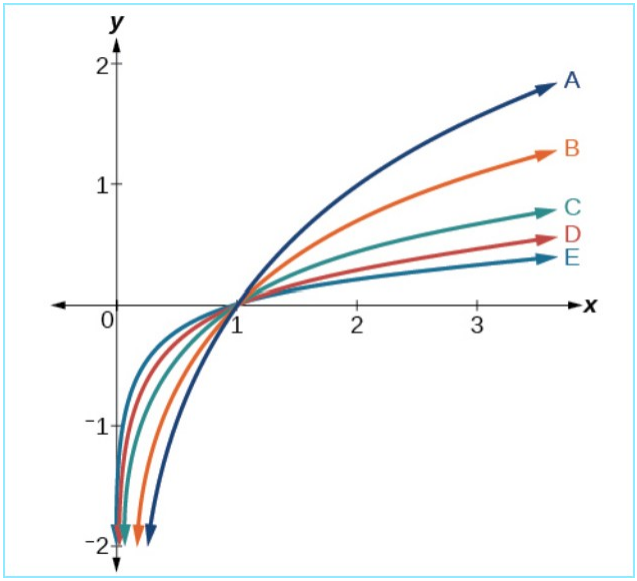
Graphs of Logarithmic Functions Videos

- Graphing a Logarithmic Function of the form $y = \log_b(x)$: Example 1
- Graphing a Horizontal Shift of a Logarithmic Function: Example 2
- Graphing a Vertical Shift of a Logarithmic Function: Example 3
- Graphing a Stretch or Compression of Logarithmic Function: Example 4
- Combining a Shift and a Strech for the Logarithmic Function: Example 5
- Combining a Shift and a Strech for the Logarithmic Function: Example 6
- Graphing a Reflection of a Logarithmic Function: Example 7

Practice Exercises

Follow the directions for each exercise below:

1. Match each function in the figure below with the letter corresponding to its graph



$d(x) = \log(x)$

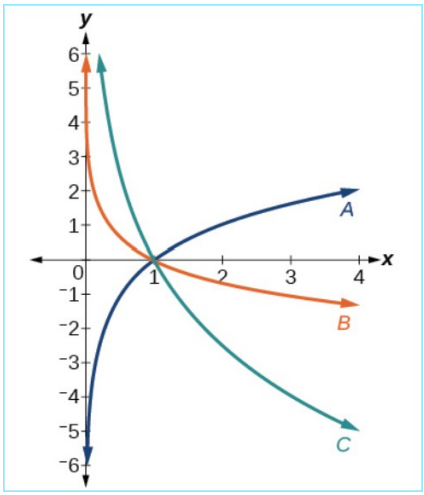
$f(x) = \ln(x)$

$g(x) = \log_2(x)$

$h(x) = \log_5(x)$

$j(x) = \log_{25}(x)$

2. Match each function in the figure below with the letter corresponding to its graph.

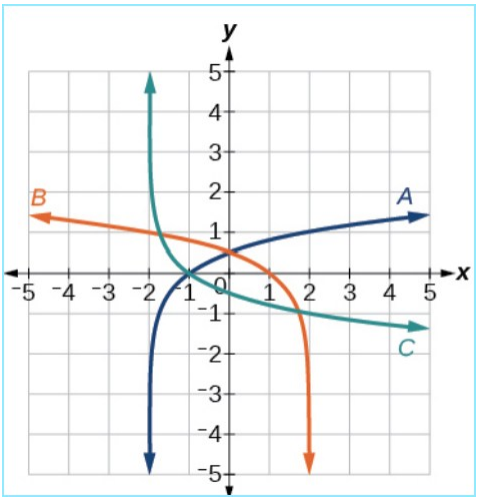


$f(x) = \log_{1/3}(x)$

$g(x) = \log_2(x)$

$h(x) = \log_{3/4}(x)$

3. Match each function in the figure below with the letter corresponding to its graph.



$f(x) = \log_4(-x + 2)$

$g(x) = -\log_4(x + 2)$

$h(x) = \log_4(x + 2)$

For the following exercises, sketch the graph of the indicated functions and state their domain, range and vertical asymptotes.

4. $f(x) = \log_2(x + 2)$

5. $f(x) = 2\log(x)$

6. $f(x) = \ln(-x)$

7. $h(x) = -\frac{1}{2}\ln(x + 1) - 3$

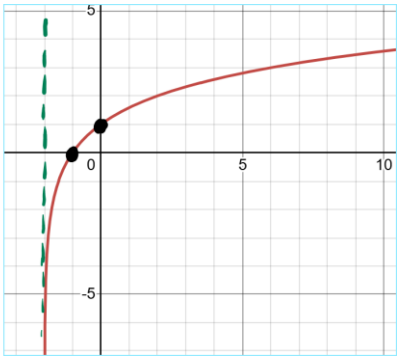
Answers:

1. $A \rightarrow g(x), B \rightarrow f(x), C \rightarrow h(x), D \rightarrow: d(x), E \rightarrow j(x)$

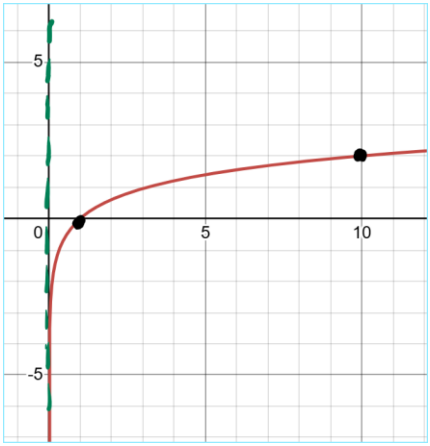
2. $A \rightarrow g(x), B \rightarrow f(x), C \rightarrow h(x)$

3. $A \rightarrow h(x), B \rightarrow f(x), C \rightarrow g(x)$

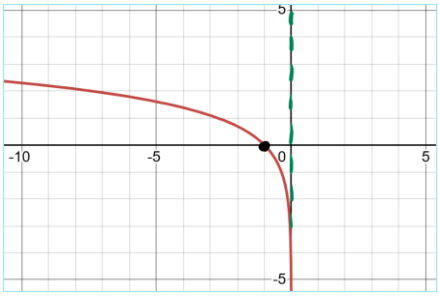
4. Domain: $(-2.\infty)$, Range: $(-\infty,\infty)$, vertical asymptote: $x=-2$



5. Domain: $(0,\infty)$, Range: $(-\infty,\infty)$, vertical asymptote: $x=0$



6. Domain: $(-\infty,0)$, Range: $(-\infty,\infty)$, vertical asymptote: $x=0$



7. Domain:$(-1,\infty)$ Range:$(-\infty,\infty)$ Vertical asymptote:$x=-1$